Comparison of Orowan Stress Models for Aluminum
Metal Matrix Nanocomposites

Abstract

Metal matrix nano-composites (MMnC) have brought much attention to the field of material science due to their enhanced mechanical properties. One of the most important mechanisms associated with the strengthening of MMnCs is the Orowan strengthening effect. The Orowan effect is well known and there are several models in literature that are specifically for MMnCs. Therefore, it is important to compare these different models in order to select the most logical model for the given conditions.

An extensive literature review was done to analyze available models describing Orowan strengthening. Two different comparisons were done, which resulted in two different average values for percent error. The first error compares the Orowan model, the simplest model available, with a more complex model found in literature. The literature model will be referred to as the Kocks model, for lack of a better term. The second relates the Orowan model, to the experimental data. The calculated errors shine light onto the applicability of the Orowan model. One error determined which model was more accurate and the other error determined the conditions under which the Orowan model can be used.

Introduction

Aluminum-based MMnCs show great promise as advanced materials for various industry sectors. These aluminum-based nano-composites have superior properties than those of typical aluminum alloys, which are regularly used by the automotive and aerospace sectors. These properties include higher strength to weight ratio, a lower thermal expansion coefficient, better wear resistance and better fatigue strength [1]. Several different nanoparticles are used as particle reinforcements in the aluminum matrix, such as carbon nanotubes (CNT), aluminum oxide (Al_2O_3), silicon carbide (SiC) and boron carbide (B_4C); all with particle sizes of about 100nm or less [2]. This paper will focus on Al_2O_3/Al nanocomposites, which can be seen in Figure 1.

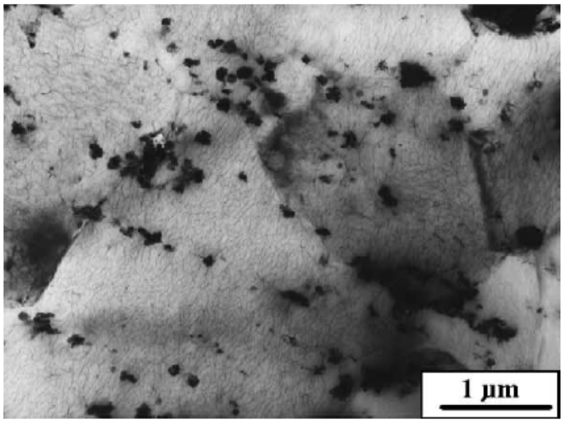

Figure 1. TEM micrographs of 1 vol.% Al_2O_3/Al [2].

Orowan strengthening can be described as the resistance to the movement of dislocations by closely packed hard particles. These particles pin the crossing dislocation and cause the dislocation to bow around them [3]. The result is a dislocation loop, or Orowan loop, which can be seen in Figure 2. These Orowan loops reduce the size of the dislocation as the dislocation moves past them. They also compresses the space in between the loops due to the added thickness of the dislocation. Figure 2 shows Orowan dislocation loops in a Ni-Fe superalloy.

Orowan looping has been extensively studied, so it comes as no surprise that there are multiple models that one can use to predict material behavior. Albeit, with such a wide range of available models, it is vital to know when it is reasonable to use each model. Since aluminum nanocomposites are some of the most widely used in industry, information on the suggested conditions for model use is of utmost importance. Here, the applicability of the Orowan equation to the Al_2O_3/Al nanocomposite is evaluated. This is done by comparing the Orowan model to the Kocks model and to experimental data found in the literature. Percent errors for both of the comparisons are calculated. The literature model was chosen on the basis of quantity of available data, simplicity of the model and material similarity.

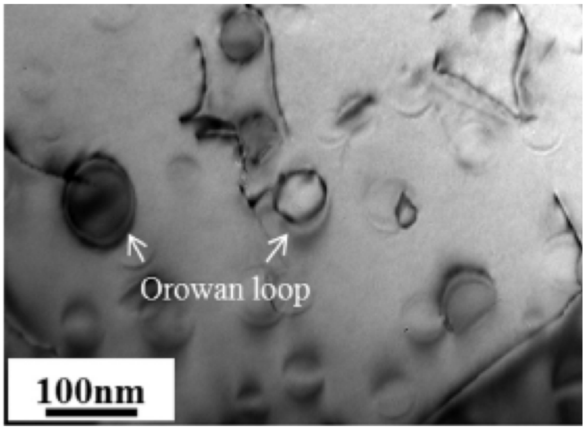

Figure 2. Orowan loops found in Metal Matrix Composites [7].

Experiment and Theory

The Orowan equation [3]

$$\sigma_{\text{Orowan}} = \frac{\text{Gb}}{\lambda} \tag{1}$$

is the simplest model found that describes the Orowan effect. In Equation 1, G is the shear modulus, b is the burger's vector and λ is the inter-particle spacing. The shear modulus and burger's vector for aluminum at 298K is 25232 MPa and 0.286nm respectively [2]. The Orowan literature equation, or the Kocks model, is described by [4]

$$\sigma_{\text{Orowan}} = \text{m}\left(\frac{0.84\text{G}}{\lambda - \text{d}}\right) \tag{2}$$

where m is the orientation factor, d is the mean particulate diameter and λ is defined as follows,

$$\lambda = \left(\frac{6\text{V}}{\pi}\right)^{\left(-\frac{1}{3}\right)} * \text{d} \tag{3}$$

In Equation (3) V is the volume fraction. The mean orientation factor, m, for randomly oriented face-centered cubic is 3.06, and the mean particulate diameter for Al_2O_3 is taken to be 50nm [2,5]. Two assumptions must be made in order to use Equation 3. The first is that the distribution of particle size is narrow, and the second is that the nanoparticles are well dispersed in the aluminum matrix [2]. Equation 3 will be used to determine the value of the Orowan stress for both the Orowan model and the Kocks model. Equation 1 can be derived with an equilibrium analysis by the principal of virtual work. The work done on the dislocation is equated to the increase in dislocation energy as follows,

$$\sigma_s b * (2\pi R) * (\Delta R) = \left(\frac{Gb^2}{2}\right) * \Delta(2\pi R) \qquad (4)$$

where σ_s is the shear stress of the dislocation. After simplification, the resulting equation is Equation 1. The related geometric parameters can be seen in Figure 3. Equation 2 and Equation 3 are derivatives from Equation 1. The Orowan equation represents the maximum shear stress that can be experienced at the dislocation.

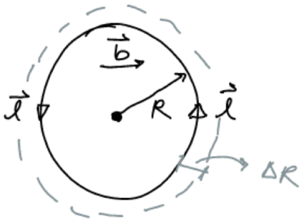

Figure 3. Geometry of a dislocation loop. *R* is the initial loop radius, *ΔR* is the change in loop radius, and *ℓ* is the line vector [9].

The literature data used for analysis can be found in Table 1 at the end of this section. These data were estimated from Figure 4. Figure 4 compares the calculated yield stresses and the experimental results that were obtained by the authors of the literature article. The aforementioned calculated stresses are beyond the scope of this paper. The dark triangle curve represents the data that are of interest.

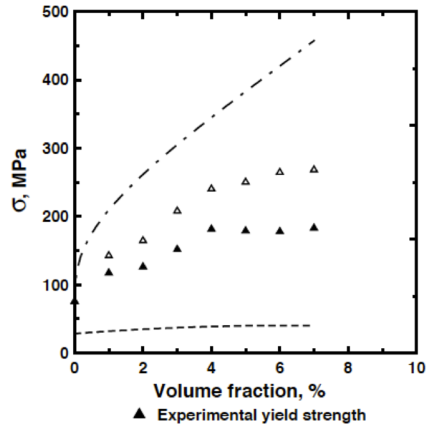

Figure 4. Experimental data ref. [2].

Table 1 shows that neither the Kocks nor the Orowan equation were able to accurately predict the yield stress that the matrix experienced, these same results can be found in graphical form in Figure 5. However, the Orowan model estimated the experimental data with a higher accuracy than the Kocks model. This suggests that the Orowan equation is, in theory, a better model to use. Nevertheless, the Orowan equation must be used with caution considering that this model still resulted in an error of over 100%. Values for a volume fraction of 0% were ignored since this volume fraction represents the pure aluminum matrix.

One explanation for the unacceptable stress predictions is that it cannot be assumed that the particulates are evenly distributed throughout the matrix. In reality, nanoparticles tend to clump together around the grain boundaries, this phenomenon can be seen in Figure 6. These nanoparticle agglomerates lead to a stress concentration, which decreases the strengthening effectiveness of the particulates [5]. Some have tried to solve this problem by applying high intensity ultra-sonic waves to the molten mixture. This has proved to increase the dispersion of nanoparticles but has not solved the problem completely [6].

Another explanation for the deviation from the true experimental values is that there are more strengthening mechanisms involved than just the Orowan stress. Such as coefficient of thermal mismatch, elastic modulus mismatch, Hall-Petch strengthening, and more. Since all of these mechanisms contribute to the overall yield stress of the nanocomposite, it is vital that all are accounted for in calculations. It is not sufficient to

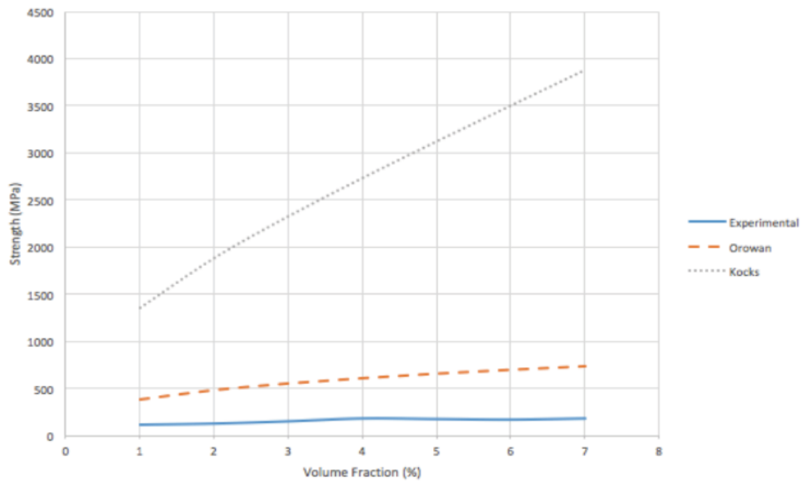

Figure 5. Graphical representation of the results.

only consider the Orowan effect in the prediction of overall yield stress in a MMnC. For example, in a paper done by a team from UW-Madison, they combine the Orowan effect, the effect of thermal mismatch, the load-bearing effect and the hall-petch effect into one equation. This resulted in a yield strength model that agreed with experimental values to a higher extent than the models shown here. However, because the focus of the paper is Orowan stress, the combination models are out of the scope of this analysis.

Finally, it is important to consider why the Orowan stress of both models is higher than the experimental stress values. The reason behind this lies in the dependence that yield stress has on particle size. Since the assumption that particles are well dispersed was not valid under these conditions, the calculated yield stress overestimates the yield stress by a factor of almost 2. In reality, the particle size is much larger since the particles tend to cluster together. It has been shown that if the volume fraction and therefore the value of λ, is adjusted to the number of “effective” particulates by means of quantitative metallography, a much better estimate of the overall yield stress is obtained [2].

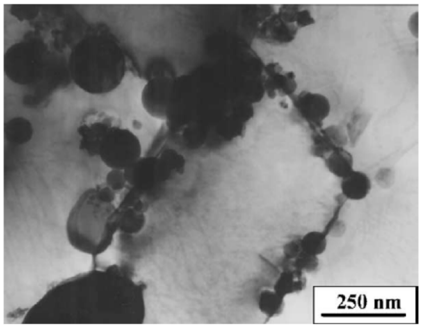

Figure 6. Agglomeration of particulates on grain boundaries [2].

Table 1. Calculated results.

Volume fraction (%)	$\sigma_{Y,Experimental}$ (MPa)	$\sigma_{s,Kock}$ (MPa)	$\sigma_{s,Orowan}$ (MPa)	Kock Error (%)	Orowan error (%)	Average Kock Error (%)	Average Orowan Error (%)
0	75	--	--	--	--		
1	120	1354	386	1028	221		
2	130	1884	486	1349	274		
3	150	2328	556	1452	271		
4	175	2735	612	1463	250		
5	170	3124	660	1738	288		
6	165	3505	701	2024	325		
7	175	3883	738	2119	322	1596	279

Summary

It is concluded from this work that the simple Orowan stress model, with a percent error of 279%, predicts aluminum nanocomposite behavior much more accurately than the Kocks model, with a percent error of 1596%. These results were not expected due to the added complexity of the Kocks model. However, the Orowan model still produced considerable inaccuracies. Therefore, it is advised that the Orowan equation only be used in a classroom setting for simple explanaition of the Orowan effect and for rough approximations of material behavior. The use of the Orowan model as a design parameter is not reasonable. In such design cases, one can find more complex models that are found in literature that are more accurate and precise than either of the two models presented here.

Metal matrix nano-composites have the potential to revolutionize many different industries with their simple concept of adding nanoparticle reinforcements to enhance material properties. The most concerning problem with these nanocomposites right now is the cost and ease of manufacturing. More studies must be done in order to fully understand the particulate diffusion behavior in liquid metal or other such phases. This will allow the control of dispersion of particulates in the metal matrix.

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